Segmentation of Diffusion Tensor Imagery *

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June 3, 2003

Abstract

Segmentation paradigms in diffusion tensor imagery (DTI) are discussed in this paper. We present a technique for determining paths of anatomical connectivity from the tensor information obtained in

*This work was supported by grants from AFOSR, NSF, ONR, and MRI.

magnetic resonance diffusion tensor measurements. The basic idea is to construct optimal curves in 3D space, where the optimality criteria is based on the eigenvalues and eigenvectors of the tensor. These curves are constructed via partial differential equations computed using multiple level-set functions. We also discuss our current efforts in clustering in DTI.

This paper is dedicated to Professor Anders Lindquist on the occasion of his 60th birthday.

1 Introduction

Fundamental advances in understanding complex biological systems require detailed knowledge of structural and functional organization in the living system. In the case of the human brain for instance, anatomical connections are related to the information pathways and how this information is processed.

During the past three decades, the neuroscience and medical communities have witnessed a tremendous growth in the field of in-vivo, non-invasive imaging of brain function. Magnetic Resonance Imaging (MRI) evolved into the modality of choice for neuroradiological examination, due to its ability to visualize soft tissue with exquisite anatomical detail and contrast.

However the resolution of MRI is well above the dimension of neural fibers and the current understanding of the nervous system is still incomplete because of the lack of fundamental connectivity information.

Diffusion Tensor MRI (DT-MRI) adds to conventional MRI the capability of measuring the random motion (diffusion) of water molecules due to intrinsic thermal agitation [2, 3]. In highly structured tissues containing a large number of fibers, like skeletal muscle, cardiac muscle, and brain white matter, water diffuse fastest along the direction that the fibers are pointing in, and slowest at right angles to it. By taking advantage of the very structure of such tissues, DT-MRI can be used to track fibers well below the resolution of conventional MRI.

Information obtained by DT-MRI consists of the diffusivities and orientations of the local principal axes of diffusion for each voxel. A wide range of techniques have been explored to provide explicit connection information from this tensor field. Early work [13] attempted to use a similarity measure to group together neighboring voxels. Other methods [4, 21] follow locally the direction of highest diffusion (this is closely related to the so called "hyper-streamline" method of [10] for tensor field visualization). In [18] a Markovian approach is used to regularize the candidate curves.

In this paper we discuss a technique to compute the anatomical paths which is based on 3D optimal curves computed via multiple level-set functions. The ideas are based on prior work on geodesic active contours [7, 9, 14], combined with numerical techniques developed in [5, 6]. The basic idea is that given two user marked points, we construct a 3D optimal-effort curve connecting these points. The effort is based on weights obtained from the diffusion tensor. The computational construction of these curves is based on representing it as the intersection of two 3D surfaces, and evolving these surfaces via the techniques developed in [5, 6]. Alternatively, one could use the work introduced in [15] for this computation. Note that the fast techniques in [12, 19, 20, 23] can not be used in the general case we discuss below due to the type of energy we use. This is in contrast with the work in [17], where the energy is artificially modified to fit the requirements for using these fast numerical approaches. It is interesting to extend the work in [9] to be able to incorporate directionality, as done below, into the penalty function, thereby permitting the use of fast numerical techniques. If the images need to be regularized prior to the geodesic computation, the approaches in [8, 18] could be used for example (see also [22] for a general theory for denoising non-flat features).

2 Active Contours and Diffusion Tensor Imaging

Once we have enhanced the DTI, e.g., via the techniques in [8, 18], we can use this to construct the flow paths (fiber tracking), e.g., for visualization. The basic idea for this is to use our prior work on geometric active contours [7, 14], as well as [9].

2.1 Brief Background on Geodesic Snakes

We briefly review some of the relevant results from [7, 14] now. We work in the plane for simplicity. All of the results extend to $\mathbf{R}^3$ as well. We first define a positive stopping term $\phi : \mathbf{R}^2 \to \mathbf{R}$ which will act as a conformal factor in the new metric we consider for our snake model. For example, the term $\phi(x, y)$ may be chosen to be small near an edge, and so acts to stop the evolution when the contour gets close to an edge. Hence, one may take

$$\phi := \frac{1}{1 + \|\nabla G_\sigma * I\|^2}, \tag{1}$$

where I is the (grey-scale) image and G_σ is a Gaussian smoothing filter.

We use ϕ to modify the ordinary Euclidean arc-length function along a curve $C = (x(p), y(p))^T$ with parameter p given by

$$ds = (x_p^2 + y_p^2)^{1/2}\, dp,$$

to

$$ds_\phi = (x_p^2 + y_p^2)^{1/2} \phi\, dp.$$

Then we want to compute the corresponding gradient flow for shortening length relative to the new metric ds_ϕ.

Accordingly set

$$L_\phi(t) := \int_0^1 \|\frac{\partial C}{\partial p}\| \,\phi\, dp.$$

Let

$$\vec{T} := \frac{\partial C}{\partial p} / \|\frac{\partial C}{\partial p}\| \ ,$$

denote the unit tangent. Then taking the first variation of the modified length function L_ϕ, and using integration by parts, we get that

$$L'_\phi(t) \quad = \quad -\int_0^{L_\phi(t)} \langle \frac{\partial C}{\partial t}, \phi\kappa\vec{N} - (\nabla\phi \cdot \vec{N})\vec{N} \rangle\, ds$$

which means that the direction in which the L_ϕ perimeter is shrinking as fast as possible is given by

$$\frac{\partial C}{\partial t} = (\phi\kappa - (\nabla\phi \cdot \vec{N}))\vec{N}.$$

As we can ignore the tangential component of the speed $\frac{\partial C}{\partial t}$ when evolving the curve C, this flow is geometrically equivalent to :

$$\frac{\partial C}{\partial t} = \phi\kappa\vec{N} - \nabla\phi \tag{2}$$

This is precisely the gradient flow corresponding to the miminization of the length functional L_ϕ. The level set, [16], version of this is

$$\frac{\partial \Psi}{\partial t} = \phi \|\nabla \Psi\| \mathrm{div}(\frac{\nabla \Psi}{\|\nabla \Psi\|}) + \nabla \phi \cdot \nabla \Psi. \tag{3}$$

One expects that this evolution should attract the contour very quickly to the feature which lies at the bottom of the potential well described by the gradient flow (3). Notice that for ϕ as in (1), $\nabla \phi$ will look like a doublet near an edge. Of course, one may choose other candidates for ϕ in order to pick out other features. This will be done for diffusion tensor images next.

2.2 Geodesic Snakes and Diffusion Tensor Imagery

For the case of DTI, we use a combination of the principal direction of the tensor with a measurement of anisotropy to define g and construct curves that will indicate the principal direction of flow (fiber tracking). Note that this can be combined with our prior work [1], where we have shown how to smoothly construct and complete curves from partial tangent data. The explicit use of a directionality constraint limits the computational techniques that can be used to find the optimal curve.

Note that in contrast with what is primarily done in the literature, when only single slices are used, we use multiple-slices (and then 3D) for these works. For this we use the computational technique developed in [5, 6], where the 3D active contour that is deforming toward the minima of the energy is represented as the intersection of two 3D surfaces. We will then need to move 3D curves, with fix end points, having the curve represented as the intersection of two deforming surfaces.

To each point in the domain $\Omega \subset \mathbf{R}^3$ we associate a 3×3 positive semidefinite symmetric matrix $A(x, y, z)$ with (real eigenvalues) $\lambda_i(x, y, z) = \lambda_i,\ i = 1, 2, 3$ and associated unit eigenvectors $\epsilon_i(x, y, z) = \epsilon_i,\ i = 1, 2, 3$. We always assume that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$.

We define the fractional anisotropy to be [17, 18]:

$$\phi(x, y, z) := \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_3)^2}}{\sqrt{2}\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}.$$

We will also consider

$$v(x, y, z) := \phi(x, y, z)\, \epsilon_1(x, y, z).$$

2.3 Diffusion Flow

In this section, we will formulate the flow which will move an arbitrary initial curve with given endpoints to a curve with respect to the weighted conformal metric defined by v. We state the results in both the plane and in space.

In what follows, we assume that we are given an embedded family of differentiable curves $C(p,t) : [0,1] \to \mathbf{R}^3$ where p is the curve parameter (independent of t), and t denotes the parameter of the family. The arc-length parametrization will be denoted by ds so that

$$ds = \sqrt{x_p^2 + y_p^2 + z_p^2}\, dp.$$

We can now state our result:

Theorem 1 *Let ϕ, ϵ_1 and v be as above. Consider the energy functional*

$$L_A(t) := \frac{1}{2}\int_0^L \|\epsilon_1 - C_s\|^2 \phi\, ds.$$

By minimizing $L_A(t)$, the following flow is obtained :

$$C_t = \phi\, C_{ss} - curl(v) \times C_s - \nabla\phi. \tag{4}$$

Proof.

We note that

$$\begin{aligned} L_A(t) &= \frac{1}{2}\int_0^L \langle \epsilon_1 - C_s, \epsilon_1 - C_s\rangle\, \phi\, ds \\ &= \frac{1}{2}\int_0^L [\langle \epsilon_1, \epsilon_1\rangle + \langle C_s, C_s\rangle - 2\langle \epsilon_1, C_s\rangle]\, \phi\, ds \\ &= \underbrace{\int_0^L \phi\, ds}_{L_A^1(t)} - \underbrace{\int \langle \epsilon_1, C_s\rangle\, \phi\, ds}_{L_A^2(t)} \end{aligned}$$

As above, we can compute that the first variation of $L_A^1(t)$ is :

$$L_A^{1'}(t) = -\int_0^L \langle \phi\, C_{ss} - \nabla\phi, C_t\rangle\, ds$$

In order to compute $L_A^{2'}(t)$, set

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = v.$$

Then since

$$L_A^2(t) = \int_0^1 (ax_p + by_p + cz_p)\, dp,$$

we have that (integrating by parts)

$$L_A^{2'}(t) = \int_0^1 [\nabla a \cdot \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} x_p + \nabla b \cdot \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} y_p + \nabla c \cdot \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} z_p - a_p x_t - b_p y_t - c_p z_t]\, dp.$$

The expression in the integral is

$$\begin{split}(b_x y_p + c_x z_p - a_y y_s - a_z z_p)x_t + (a_y x_p + c_y z_p - b_x x_p - b_z z_p)y_t + \\ (a_z x_p + b_z y_p - c_x x_p - c_y y_p)z_t.\end{split} \quad (5)$$

Noting that

$$\text{curl}(v) = \begin{pmatrix} c_y - b_z \\ a_z - c_x \\ b_x - a_y \end{pmatrix},$$

we see that we may write (5) as

$$-\langle \text{curl}(v) \times \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix}, C_t \rangle,$$

and so

$$L_A^{2'}(t) = -\int \langle \text{curl}(v) \times \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix}, C_t \rangle\, ds.$$

Since

$$L_A'(t) = L_A^{1'}(t) - L_A^{2'}(t),$$

the theorem follows. QED

Remarks:

1. The above energy is minimum when the tangent of the curve C is most closely aligned with the direction of the principal eigenvector ϵ_1. Moreover this constraint is weighted by the anisotropy ϕ defined previously. When A is almost isotropic, we have $\lambda_1 \approx \lambda_2 \approx \lambda_3$. In this region, $\phi \approx 0$ ensures that we will not penalize a curve that would not be perfectly aligned with ϵ_1 (which here cannot be considered the unique direction of diffusion). On the other hand, if $\lambda_1 \gg \lambda_2 \geq \lambda_3$ there is no ambiguity : ϵ_1 is the preferred direction for diffusion and $\phi \approx 1$ ensures that the curve will follow closely.

2. In two dimensions, we can consider that $c = 0$ and $a_z = b_z = 0$. As is standard we define $\text{curl}_{2\text{D}}$ to be the scalar $b_x - a_y$.

   Therefore the following relation holds :

$$\begin{aligned} \text{curl}_{3\text{D}}(v) &= \begin{pmatrix} 0 \\ 0 \\ b_x - a_y \end{pmatrix} \\ &= \text{curl}_{2\text{D}} \begin{pmatrix} a \\ b \end{pmatrix} e_z \end{aligned}$$

   Since $e_z \times \vec{T} = \vec{N}$ ($\vec{T}$ is the unit tangent and $\vec{N}$ the unit normal), we get the flow

$$C_t = (\phi\kappa - \langle \nabla\phi, \vec{N} \rangle - \text{curl}_{2\text{D}}(v))\vec{N} \tag{6}$$

   Note that by the standard Frenet formulas in the plane

$$C_{ss} = \kappa\vec{N},$$

   so that equations (4) and (6) are consistent.

3. The above curve deformation is implemented in 3D deforming the intersecting of two 3D surfaces. In addition, the end points of the deforming curve are fixed.

4. In case an advanced initialization is needed, we can use for example the technique in [17]. We are also investigating the use of the geodesics obtained from just $\int \phi ds$, which can be computed using fast numerical techniques, to initialize the flow.

5. The above described technique can be used for finding discrete connectivity lines. We are currently also working on the use of techniques such as those in [11] to cluster the diffusion direction information.

3 Conclusions

In this paper, we discussed a geodesic snake technique for the segmentation of diffusion tensor imagery. DTI is an increasing important non-invasive methodology which can reveal white matter bundle connectivity in the brain. As such it is useful in neuroscience as well as image guided surgery and therapy.

Using the conformal metric ideas we derived an explicit flow for the segmentation of such imagery in three dimensions. In future work, we plan to test our flow on some explicit examples. Level set ideas will of course be very important in the computer implementation and the development of fast reliable algorithms based on the equation (4).

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